AN ESTIMATE OF THE EFFECT OF BACK SPIN ON THE CRITICAL ANGLE IN THE UNYAWED RICOCHET OF A CYLINDER

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A method is described for giving an estimate of the effect of back spin on the critical angle for the ricochet of a cylinder off water. The method incorporates both the spin and the weight of the projectile into the Birkhoff *et al* theory, as described by Johnson and Reid (1)‖ for a sphere. The differences and predictions concerning hydrodynamic pressure and lift, etc., are compared with those made by several other authors. A comparison of our new results with some available experimental results has also been made.

1 INTRODUCTION

The ricochet of solid projectiles off water at certain impact angles is a mechanical phenomenon that is both highly amusing and highly serious. In the second World War, Sir Barnes Wallis found the phenomenon basic to his design of the bouncing bomb. In particular, he introduced back spin at the time of release of the bomb in order to enhance its chance of success in attacking the Möhne dams (2)(3).

It is known that back spin can increase the critical impact angle for successful ricochet, which increases the probability of a bouncing bomb hitting its target. However, there is no wholly satisfactory theory for predicting the influence of back spin. Johnson and Reid in their review (1) applied the Birkhoff theory to the ricochet of a spherical projectile off water, and demonstrated that it is in good agreement with the known empirical expression for critical impact angle

$$\theta_c = 18°/\sqrt{\sigma} \tag{1}$$

σ is the projectile specific gravity. They did not, however, show how to incorporate spin into the theory.

Hutchings, in his paper (4), argued that the Birkhoff theory cannot predict that spin will have any effect on the tendency of a sphere to ricochet and that it is incapable of explaining the benefit which Wallis appears to have gained by imparting spin. Accordingly, the Birkhoff pressure expression

$$p = \tfrac{1}{2}\rho u^2 \cos^2 \beta \tag{2}$$

was assumed to be physically *un*realistic; p denotes pressure, ρ density of water, u the projectile velocity, and β the angle between the normal to the surface element and the velocity vector.

In this paper a comparison is made of Johnson and Reid's approach, referred to as the Birkhoff theory, with that of Hutchings. The focus of attention is on Hutchings' assumptions about the pressure distribution formula and the 'wetted' area of the projectile. It may be shown that the velocity combination rule adopted by Hutchings, which is valid kinematically, is not suitable for dynamic pressure calculations. As based on the former it was asserted that the Birkhoff formula was incapable of explaining the spin effect. By utilizing an exact, ideal, incompressible fluid flow solution and the empirical Birkhoff formula, the present paper establishes an approximate pressure expression from which a formula is obtained for the critical impact angle of ricochet. Both spin effect and the weight of the projectile are included in the latter formula. Some discussion and estimates of the magnitude which spin effects make are presented below.

1.1 Notation

a	Radius of cylinder
g	Gravitational acceleration
p	Pressure
r, θ	Cylindrical coordinates
t	Time
u	Projectile velocity
v	Flow velocity
β	Angle between normal to surface element and velocity of projectile
θ_c	Critical impact angle
ρ	Density of water
σ	Specific gravity of projectile
ϕ	Angular variable
Φ	Flow potential
ω	Angular velocity
L	Lift

2 BASIC ASSUMPTIONS

For calculating the pressure exerted on the surface of a submerged cylinder, instead of the formula (2) proposed by Birkhoff *et al.* (1), Hutchings (4) introduced Rayleigh's expression

$$p = \frac{\pi \cos \beta}{4 + \pi \cos \beta} \cdot \rho u^2 \tag{3}$$

but modified it to

$$p = \frac{\pi \cos \beta}{5} \rho u^2 \tag{4}$$

Rayleigh's formula is for the mean pressure on a flat lamina in an oblique stream (see Fig. 1). Lacking an exact formula for the pressure distribution on the surface of a cylinder partly submerged in water, Hutchings' approach was commendably novel.

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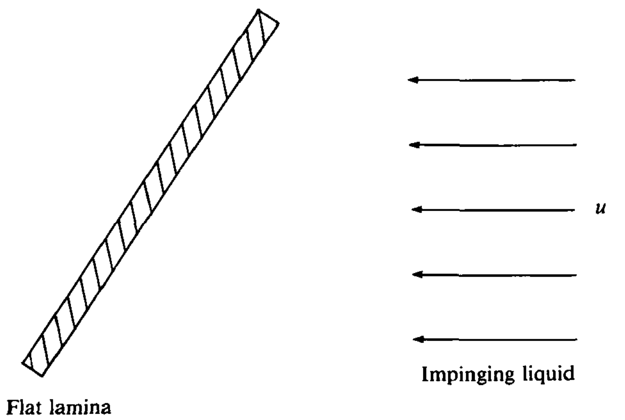

Fig. 1. A representation of the flow field for the Rayleigh formula (3)

Furthermore, Hutchings used the combination rule for velocities

$$(u')^2 = (u + a\omega \cos \phi)^2 + (a\omega \sin \phi)^2 \quad (5)$$

where u' denotes the total velocity of a surface element, u the translational component, ω the angular velocity, a the radius of the cylinder, and ϕ the angle shown in Fig. 2. Also, this rule was applied to the pressure formula (2) assuming $a\omega/u$ to be small and the second order quantity, $(a\omega/u)^2$, negligible; the spin itself caused no change in lift (see Fig. 2). Thus it was asserted that the Birkhoff theory was incapable of explaining spin effect. However, as will be shown in the next section this reasoning is questionable.

When Hutchings combined the velocity combination rule (5), with the corresponding change in angle, β, into his pressure formula (4), he found it to underestimate the lift of a fully submerged cylinder due to spin. The lift he gave was

$$\tfrac{2}{15}\pi\rho a^2 \omega u \quad (6)$$

whereas the exact solution of fluid dynamics (**4**) is

$$2\pi\rho a^2 \omega u \quad (7)$$

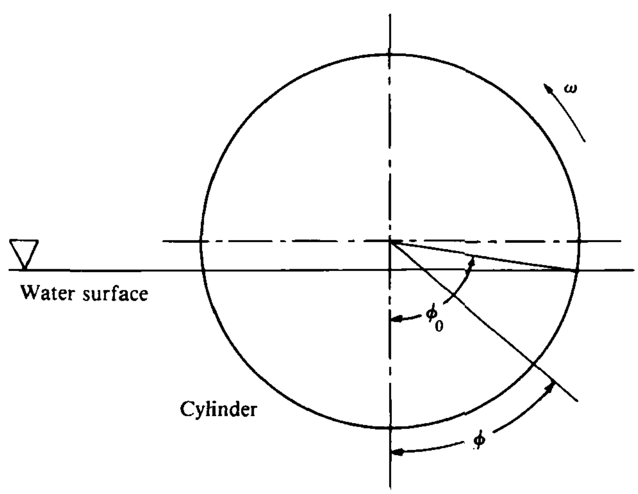

Fig. 2. The plan view of a partially submerged cylinder defining angles ϕ and ϕ_0

Even though only the leading surface was taken into account in his analysis, requiring a doubling of the value in (6), the lift due to spin given by Hutchings is still only one eighth that in (7).

Another difficulty arises concerning the so-called 'wetted' area. Hutchings supposed that the 'wetted' area, or the area over which pressure was exerted by the fluid was $2\phi_0$, not ϕ_0 (see Fig. 2).

His argument was that for a cylinder impinging obliquely on water, the pressure exerted must act over a considerable area because of the splash created. But from ricochet photographs the splash is evidently very different in character from that of the fluid beneath the surface. Some authors have specifically remarked that the pressure in the splash is effectively atmospheric (**5**).

By adopting $2\phi_0$ rather than ϕ_0 a pressure and lift is found by Hutchings' approach which is higher than that given by Birkhoff using just the angle ϕ_0 (see Figs 3 and 4). Roughly speaking, the lift given by Hutchings' assumptions is about double that given by Birkhoff *et al.*

In summary, the essence of ricochet analysis turns less on how to define the relationship between wetted area and pressure than on the choice of the pressure formula.

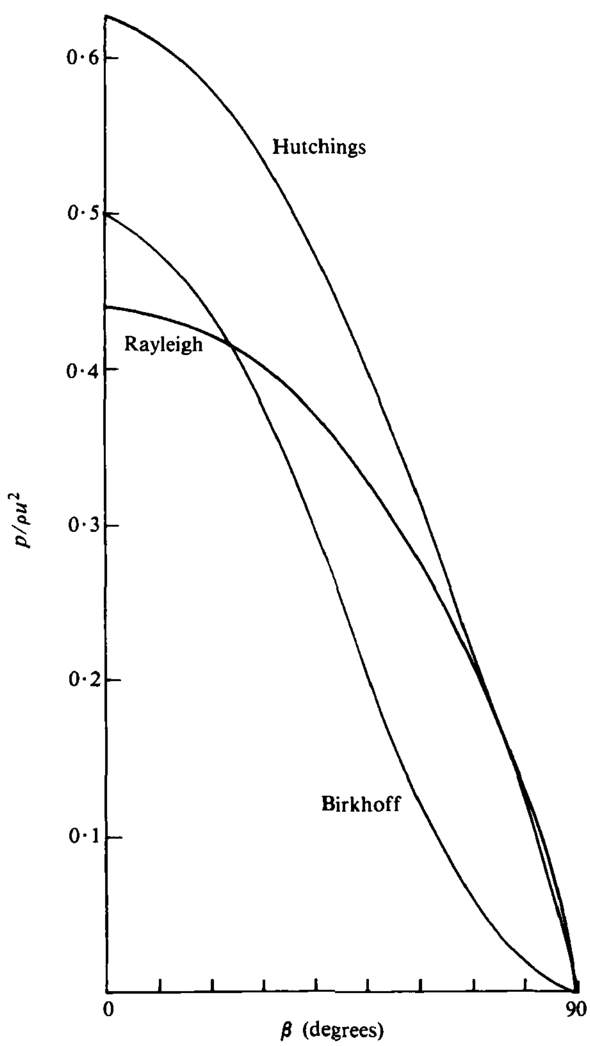

Fig. 3. Dimensionless pressure versus β, the angle between the normal to a surface element and the velocity of the projectile

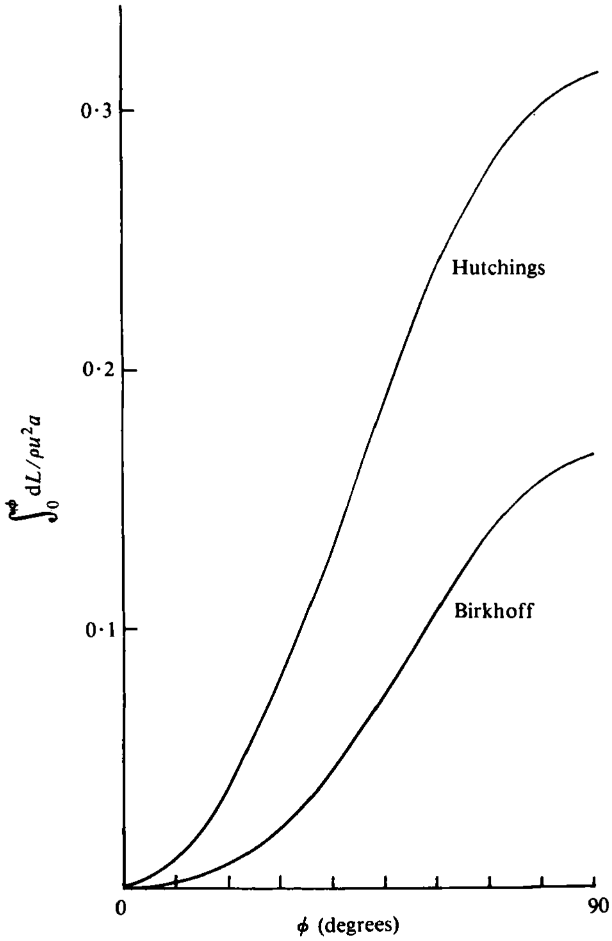

Fig. 4. Dimensionless lift integral versus angle ϕ

Based on the above arguments, an alternative approach is proposed below. The basic assumptions involved are:

(1) ideal incompressible fluid,
(2) a wetted angle over which the pressure acts of ϕ_0,
(3) a pressure exerted on the surface of a submerged translationally moving cylinder described by (2).

3 THE BASIC EQUATION

In this section we first outline the solution for flow over a cylinder, moving with a velocity u perpendicular to its length and rotating with angular velocity ω around its axis (6). We have,

$$\bar{v} \cdot \bar{n}_r \Big|_{r=a} = \bar{u} \cdot \bar{n}_r \tag{8}$$

and

$$\bar{v} \cdot \bar{n}_\theta \Big|_{r=a} = \bar{u} \cdot \bar{n}_\theta + \omega a \tag{9}$$

where $\bar{v}$ is the mean flow velocity vector, r and θ the polar coordinates, and $\bar{n}$ is the vector normal to the surface of the cylinder. From potential flow theory the potential, Φ, and velocity, $\bar{v}$, are

$$\Phi = -\frac{a^2\bar{u} \cdot \bar{n}_r}{r} + \{\omega a^2 + 2a(\bar{u} \cdot \bar{n}_\theta)\}\theta \tag{10}$$

and

$$\bar{v} = \text{grad } \Phi = \left(\frac{a}{r}\right)^2 (\bar{u} \cdot \bar{n}_r)\bar{n}_r + \left\{-\left(\frac{a}{r}\right)^2 (\bar{u} \cdot \bar{n}_\theta) + \frac{a}{r}(\omega a + 2\bar{u} \cdot \bar{n}_\theta)\right\}\bar{n}_\theta \tag{11}$$

Because the fluid at infinity is at rest, Cauchy's theorem leads to

$$\frac{p}{\rho} = -\frac{v^2}{2} - \frac{\partial \Phi}{\partial t} \tag{12}$$

Considering the moving coordinates to have the same velocity $\bar{u}$, then

$$\frac{\partial \Phi}{\partial t} = \frac{\mathrm{D}\Phi}{\mathrm{D}t} - \bar{u} \cdot \text{grad } \Phi \tag{13}$$

$$= -\frac{a^2}{r}\left(\frac{\mathrm{D}\bar{u}}{\mathrm{D}t}\right) \cdot \bar{n}_r + \left\{a^2 \frac{\mathrm{D}\omega}{\mathrm{D}t} + 2a\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}t} \cdot n_\theta\right)\right\}\theta - \bar{u} \cdot \bar{v} \tag{14}$$

where D/Dt refers to Lagrangian differentiation. By combining (12) and (14), the pressure acting on the surface of the cylinder moving with constant translational velocity, u, and angular velocity, ω, becomes

$$\frac{p}{\rho} = \bar{v} \cdot \left(\bar{u} - \frac{\bar{v}}{2}\right)\Big|_{r=a} = \frac{u^2}{2} - \frac{1}{2}(2u \sin \theta + \omega a)^2 \tag{15}$$

When $\omega a \ll u$, this pressure can be expressed as

$$\frac{p}{\rho} = \frac{u^2}{2} - 2u^2 \sin^2 \theta - 2u\omega a \sin \theta \tag{16}$$

At this point it is worth noting the unsuitability of the velocity combination rule for determining the dynamic pressure expression in a fluid. Principally, it is that in any dynamic pressure expression—that of either Bernoulli or Rayleigh—the velocity, u, is treated as the upstream velocity or the translational velocity of the object. If there is some change in the velocity of the fluid or object, only the corresponding potentials can be linearly superposed and then used to calculate the new velocity field and pressure according to the new velocity field. If in an infinite fluid the translation and spin are combined by a velocity combination rule and the pressure formula for the translational movement is used, the result is different from the exact expression (15).

In the case of a fully submerged cylinder the total pressure, consisting of both translation and spin effects, is expressed in (16). Reverting to (15), it is seen that the term $2u \sin \theta$ couples the translation and spin effects. In order to obtain a simple approximate formula appropriate to a partially submerged cylinder, it is assumed in the present case that there is an undetermined coupling term instead of $2u \sin \theta$. When $a\omega \ll u$, the pressure expression becomes

$$\frac{p}{\rho} = \frac{u^2}{2} - \frac{1}{2}(c + \omega a)^2$$
$$\simeq \frac{u^2}{2} - \frac{c^2}{2} - c\omega a \qquad (17)$$

Supposing that the formula (2) applies for a partially submerged cylinder without spin, comparing (2) and (17) then gives

$$c^2 = u^2 \sin^2 \theta \qquad (18)$$

Thus, the pressure expression becomes

$$\frac{p}{\rho} = \frac{u^2}{2} \cos^2 \theta - \omega a u \sin \theta \qquad (19)$$

(θ is the angle between the normal to the surface and the velocity vector, u.)

The lift for a just fully submerged cylinder, from (19), is

$$\tfrac{1}{2}\pi \rho a^2 \omega u \qquad (20)$$

which is about four times Hutchings' value, but still only one half the exact value (7) for infinite immersion, when adding the rear surface effect.

4 THE CRITICAL RICOCHET EQUATION

Using the pressure formula (19), the lift $\mathrm{d}L$ on an arc element $\mathrm{d}l$ is

$$\mathrm{d}L = p \cos \phi \, \mathrm{d}l$$
$$= \rho\left(\frac{u^2}{2} \sin^2 \phi + \omega a u \cos \phi\right) \cos \phi \, \mathrm{d}l \qquad (21)$$

supposing that the impact angle is very small or that the velocity, u, is nearly parallel to the horizontal. If the wetted area over which pressure applies is defined as $(0, \phi_0)$ (see Fig. 2), then the lift exerted on unit length of cylinder becomes (Fig. 4)

$$L = \int_0^{\phi_0} \mathrm{d}L$$
$$= \frac{a\rho u^2}{6} \sin^3 \phi_0 + \rho \omega a^2 u\left(\frac{\phi_0}{2} + \frac{\sin 2\phi_0}{4}\right) \qquad (22)$$

Taking the weight of the projectile into account, the vertical motion of the cylinder is governed by

$$-\rho' \pi a^2 \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = L - \rho' \pi a^2 g \qquad (23)$$

where ρ' denotes the density of the projectile and g is the gravitational acceleration.

By following the procedure given in (2) and supposing that the projectile velocity remains constant during ricochet, integration of (23) gives the critical impact angle as

$$\theta_c^2 = \theta_{c0}^2\left(1 + 8\,\frac{\omega a}{u}\right) - \frac{4ag}{u^2} \qquad (24)$$

with

$$\theta_{c0}^2 = \frac{1}{8\sigma} \qquad (25)$$

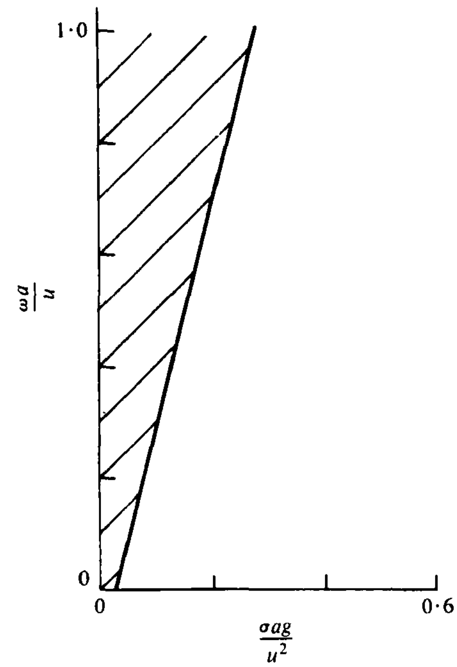

Fig. 5. Inter-relationship between back spin, weight, cylinder radius, and velocity of projectile for ricochet

the critical impact angle without spin. It is obvious from (24) that back spin would increase the critical impact angle and top spin decrease it.

For comparison, Hutchings results are

$$\theta_c^2 = \theta_{c,H}^2\left(1 + \frac{11}{7}\frac{a\omega}{u}\right) \qquad (26)$$

and

$$\theta_{c,H}^2 = \frac{8}{75\sigma} \qquad (27)$$

the latter is the critical impact angle without spin given by Hutchings. Seemingly (24) predicts a much greater spin effect than does (26).

Figure 5 shows the relationship between weight, back spin, and velocity of projectile for critical ricochet. In the shaded area, i.e.

$$\frac{\omega a}{u} > \frac{4\sigma a g}{u^2} - \frac{1}{8} \qquad (28)$$

critical ricochet can occur, and the heavier the projectile, the larger the back spin contribution needed for critical ricochet. A given spin and weight are more important the lower the impact speed.

5 EXPERIMENTAL COMPARISON

There appears to be no exact measurement available of the spin effect in ricochet, so that it is hard to make a sound judgement about the validity of the formulae previously proposed. From certain practical circumstances evidence is, however, available and sheds some light on this particular issue. Hence, in this section two examples are examined and the effect of spin is estimated using our proposed formula and comparing it with the practical results available.

The first example pertains to Wallis's bouncing bomb with the data quoted in (4). Slightly simplified, the diameter and density of the bomb are taken to be 1·27 m and 2·17 g/cm^{-3}, respectively, the velocity approximately 110 m/s^{-1}, the minimum impact angle 10 degrees, and the spin at release of the bomb 52 rad/s^{-1}, i.e., about 8·5 revolutions per second. From this data it is clear that the weight of the bomb can be ignored. If there was no spin, equation (25) would give $\sim$ 13·5 degrees as the critical impact angle. Considering air resistance and the roughness of the sea surface, and since 10 degrees was the minimum impact angle, it was necessary to make use of back spin at the time of release of the bomb. 52 rad/s^{-1} back spin and 110 m/s^{-1} projectile velocity give the term $(\omega a)/u \sim 1/3$. Hutchings obtained 16 degrees as the critical impact angle with equation (26) whereas (24) gives this critical impact angle as 25 degrees. The latter result thus suggests that the chance of a successful ricochet is improved and that a bomb could successfully attack dams.

Another example concerns an experiment comparing no spin and uncontrolled spin tests carried out by Soliman, Johnson, and Reid (7). In their tests it was shown that a steel spherical projectile ricochets off water with no spin or uncontrolled spin. They found about 1 degree difference in critical impact angle at about 6 degrees and about 160 ft/s projectile velocity. If the uncontrolled top spin peripheral velocity is one or two orders of magnitude less than the projectile velocity, then the spin would be

$$\text{(i)} \quad \text{if } \frac{\omega a}{u} \sim 10^{-2}, \text{ then } \omega \sim 100 \text{ rad/s}^{-1} \sim 16 \text{ revs/s} \qquad (29)$$

and

$$\text{(ii)} \quad \text{if } \frac{\omega a}{u} \sim 10^{-1}, \text{ then } \omega \sim 1000 \text{ rad/s}^{-1} \sim 160 \text{ revs/s} \qquad (30)$$

Obviously both offer the prospect of a significant degree of spin, especially the latter. The differences in critical angles for non-spin and uncontrolled spin projectile should be, if $\omega a/u \sim 10^{-2}$

$$\text{from (26);} \quad \frac{\Delta\theta_c}{\theta_c} \sim \frac{11}{14}\frac{a\omega}{u} \sim 10^{-2} \qquad (31)$$

$$\text{and from (24);} \quad \frac{\Delta\theta_c}{\theta_c} \sim \left(\frac{\theta_{c0}}{\theta_c}\right)^2 4\,\frac{a\omega}{u} \sim 4 \cdot 10^{-2} \qquad (32)$$

These compare with a difference in observed critical impact angle of 17 per cent, i.e., 1 degree in 6 degrees.

6 CONCLUSIONS

To determine the conditions under which rigid projectiles ricochet from a liquid surface leads to a difficult impact problem in unsteady flow fluid mechanics. Though the phenomenon itself is well known and has been employed militarily for at least some centuries (1), theory to account adequately for the interrelationships of the various parameters has only been achieved recently. Previous investigators have usually been concerned with the effects of the translational speed of the projectile and only the paper by Hutchings endeavours to theoretically assess the effect of rotational speed. The effects of spin and weight on projectile performance can be large, as was shown by Barnes Wallis's spinning bouncing bomb, and thus these specific factors are indeed important.

The work described above assesses the effect of rotation and weight and results are derived which are believed to be more soundly based than those of Hutchings; for this reason they should be more reliable than his. We have endeavoured to test the validity of our expressions against such meagre physical evidence as exists and believe it shows them to credibly represent observed behaviour. However, detailed experimental work on ricochet aimed particularly at elucidating the effects of spin and weight and seeking to assess the accuracy of all the various expressions now given is necessary. This will represent a difficult investigation, however, but would appear to be the essential next step if the subject is to be advanced.

APPENDIX

REFERENCES

(1) JOHNSON, W. and REID, S. R. 'Ricochet of spheres off water', *J. mech. Engng Sci.*, 1975, **17**, 71–81.
(2) BRICKHILL, P. *The Dam Busters*, 1966 (Evans, London).
(3) MORPURGO, J. E. *Barnes Wallis*, 1972 (St. Martin's Press, New York).
(4) HUTCHINGS, I. M. 'The ricochet of spheres and cylinders from the surface of water', *Int. J. mech. Sci.*, 1976, **18**, 243–247.
(5) SHIFFMAN, M. and SPENCER, D. C. AMP Report 42. 4R, AMG-NYU No. 133, July (1945).
(6) LAMB, H. *Hydrodynamics*, 1932 (Cambridge University Press).
(7) SOLIMAN, A. S., REID, S. R., and JOHNSON, W. 'The effect of projectile speed in ricochet off water and sand', *Int. J. mech. Sci.*, 1976, **18**, 279–284.