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Initial Development of Microdamage Under Impact Loading[1]

In this paper, the initial development of microdamage in material subjected to impulsive loading was investigated experimentally and analytically with controllable short-load duration. Based on a general solution to the statistical evolution of a one-dimensional system of ideal microcracks, a prerequisite to experimental investigation of nucleation of microcracks was derived. By counting the number of microcracks, the distribution of nucleation of microcracks was studied. The law of the nucleation rate of microcracks can be expressed as a separable function of stress and cracksize. It is roughly linear dependence on loading stress. The normalized number density of microcracks is in agreement with that of a second-phase particle.

1 Introduction

Spallation, occurring in solids subjected to impact loading, usually results from accumulation of microdamage. Generally speaking, the microdamage is created by tensile stress waves, which form when compressive waves reflect at free surface, corners, or interfaces adjacent to media with low-wave impedance. Closeup observations have revealed that the microdamage is produced by means of nucleation, extension, and coalescence of microcracks or microvoids (Curran et al., 1987). The idea that coalescence of microcracks or microvoids should be responsible for complete spallation was suggested long ago. However, the evolution of microdamage, especially the transition from gradual accumulation of microdamage to complete failure of materials, has not been clearly interpreted yet, either experimentally or theoretically.

In a previous paper (Shen et al., 1986) it has been shown that the collapse of residual strength of damaged samples appears to be catastrophic at a certain level of microdamage. The specimens were cut from rolled aluminum alloy plate and tested under planar impact loading with a light gas gun. Then the central part of a half of an individual impacted specimen was statically tested to examine the residual ultimate strength of the damaged sample. Another half was sectioned, polished, and observed with a microscope to investigate corresponding microdamage. It seems that an abrupt loss of residual ultimate strength happens at $l'/l \sim 0.7$, where l' is the total length of microcracks adjacent to a would be separation line and l is the length of the observed section (Fig. 1). In this diagram the damage function is defined by $F = 1 - \sigma_r/\sigma_b$, where σ_r and σ_b are the residual ultimate tensile strength of impacted sample and the bulk strength of the virgin material, respectively. Clearly, the sharp loss of residual ultimate strength of damaged material manifests a critical state of microscopic damage. In this regard damage fracture transition represents a class of material instability.

For the sake of understanding the instability, it is necessary to determine the variables which can properly characterize the accumulation of microdamage. As the basis of the study, this paper restricts consideration to the initial development of microdamage, i.e., the nucleation of microcracks under planar impact loading, because spallation occurring under this condition causes planar penny-shaped microcracks parallel to each other. Thus, the configuration can simplify the problem as one-dimensional, since only one variable is needed to characterize the microcracks.

2 General Solution

A general framework concerning the statistical evolution of microdamage has been put forward in previous papers (Bai et al., 1988). Here, a brief introduction will be given, particularly for the case of a one-dimensional system of ideal microcracks.

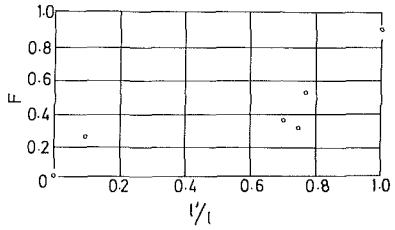

Fig. 1 Relationship between macroscopic damage function $F = 1 - \sigma_r / \sigma_b$ and total length of microcracks by observed section length l'/l (from Shen et al., 1986)

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Microcracks are termed as ideal provided they satisfy the following conditions: (1) nucleation and extension of microcracks are independent of each other and (2) nucleation and extension of an individual microcrack are governed by its microscopically local conditions. In addition, it is assumed that each microcrack can be characterized by a single variable in phase space. For example, a penny-shaped crack can be described by its area or radius. Obviously, this assumption can significantly simplify the formulation of laws of nucleation and extension of microcracks.

The governing equation of the statistical evolution of a one-dimensional system of ideal microcracks has been derived (Bai et al., 1988; Ke et al., 1990; Bai et al., 1991) in a phase space and can be written as

$$\frac{\partial n}{\partial t}+\frac{\partial(\dot{c}n)}{\partial c}=n_N \tag{1}$$

where t is time; c is the length scale variable of microcrack; $\dot{c}$ is the extension rate of an individual crack; n is the number density of cracks, i.e., the number of cracks per unit physical volume per unit crack length; and n_N is nucleation rate of number density of cracks, i.e., nucleating number of cracks per unit physical volume per unit crack length per unit time. The details of the derivation of Eq. (1) can be found in the paper of Ke et al. (1990) and Bai et al. (1991). Clearly, the dynamic laws for n_N and $\dot{c}$ are dependent on loading stress $\sigma(t)$ and material properties in addition to the microcrack variable c, but are independent of the number density n, in the system of ideal microcracks:

$$n=n(c,\ \sigma(t),\ X_m) \tag{2}$$

$$\dot{c}=\dot{c}(c,\ \sigma(t),\ X_m) \tag{3}$$

where X_m are material parameters.

The general solution to Eq. (1) has been obtained by Ke et al. (1990) and expressed in the following form:

$$n(c,\ t)=\begin{cases} n_N(c)t & c\le b \\ \dfrac{1}{A(c)}\displaystyle\int_{\eta(c,\ t)}^{c} n_N(c')dc' & c>b \end{cases} \tag{4}$$

provided the loading stress σ remains constant. Hence, σ and X_m are not denoted explicitly in (4). Here, $\dot{c}$ is defined by

$$\dot{c}=\begin{cases} 0 & c\le b \\ A(c) & c>b \end{cases} \tag{5}$$

where b denotes a size threshold of extension of microcrack and η is defined in the following way:

$$t=\int_{\eta(c,\ t)}^{c}\frac{dc'}{A(c')}. \tag{6}$$

When $c \Rightarrow b$, the asymptotic behavior of extension rate $A(c)$ determines whether a stationary solution exists in the range of $b<c<c_0$, where c_0 is defined by

$$t=\int_{b}^{c_0(t)}\frac{dc'}{A(c')}.$$

The stationary solution to Eq. (1) manifests the saturation of the number density of microcracks (Ke et al., 1990). But in this paper we have to focus our discussion on the nucleation of microcracks. The readers, interested in the theoretical detail of the evolutionary solution to Eq. (1), can refer to the Ke et al. (1990) paper.

3 Experimental Procedure and Distribution Function

It was pointed out in the previous section that two dynamic laws, nucleation and extension of microcracks, can substantially affect the evolution of microcracks. There is a simple extension law derived by Berry (1960) for a crack in a linear elastic medium. Of course one cannot expect that the microcracks in the micrometer range are truly brittle. But before realistic models of microcracks are developed, Berry's formula can be adopted as an operational expression of an extension law.

On the other hand, nucleation laws of microcracks proposed hitherto are mostly indirect (Curran et al., 1987; McClintock, 1973; Batdorf, 1975). Due to the significance of the function n_N in the evolution of microcracks, for example in expression (4), the determination of the nucleation function was thought to be a primary task in the experimental study. But extracting the information on the nucleation of microcracks from experimental observations is difficult because one cannot observe straightforwardly the nucleation of microcracks. In order to unveil the nucleation law of microcracks let us examine the solution (4)–(6) of the evolution of microcracks. The concrete aim is to guide the design of experiments.

For a very short stress pulse $\alpha(\delta t)$, the expression (6) can be rewritten as

$$\dot{\eta}=c-A(c)\cdot\delta t. \tag{7}$$

In fact, $\eta=\eta(c,\ t)$ represents the size of microcracks at $t=0$, for the crack of length c at time t (Ke et al., 1990), if it could contract according to the same extension law (5). Substitution of (7) into (4) gives following approximate solution for a short stress pulse,

$$n(c,\ \delta t)=\begin{cases} n_N(c)\cdot\delta t & c<b \\ n_N(c-\theta\cdot A(c)\cdot\delta t)\cdot\delta t & c>b \end{cases} \tag{8}$$

where θ is a parameter $0<\theta<1$.

If we intend to express the nucleation rate $n_N(c)$ as

$$n_N(c)\doteq\frac{n(c,\ \delta t)}{\delta t}, \tag{9}$$

the following inequality should be satisfied:

$$c>>\theta\cdot A(c)\cdot\delta t. \tag{10}$$

The typical extension rate of microcrack $A(c)$ could be estimated by observing the length scale of microcracks in the

Nomenclature

$A(c)$ = extension rate of individual crack when $c>b$
b = threshold of extension of microcrack
c = length scale variable of microcrack
$\dot{c}$ = extension rate of microcrack
n = number density of microcrack, i.e., number of microcracks per unit physical volume per unit crack length
n_N = nucleation rate of number density of microcracks, i.e., nucleating number of cracks per unit physical volume per unit crack length per unit time
N = total number of microcracks per unit volume
t = time
X_m = material parameters
ρ = normalized number density of microcracks
σ = stress

Subscripts

p = variable on sectioned surface
N = variables describing nucleation of microcracks

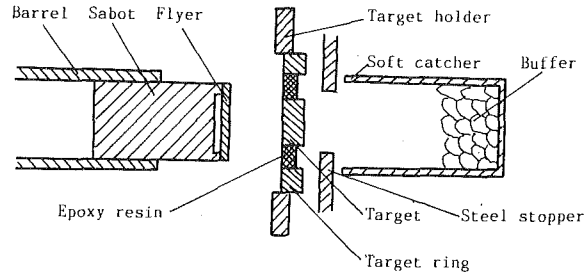

Fig. 2 Set up of light gas gun

Fig. 3 Microcracks formed in specimen under short stress pulse loading

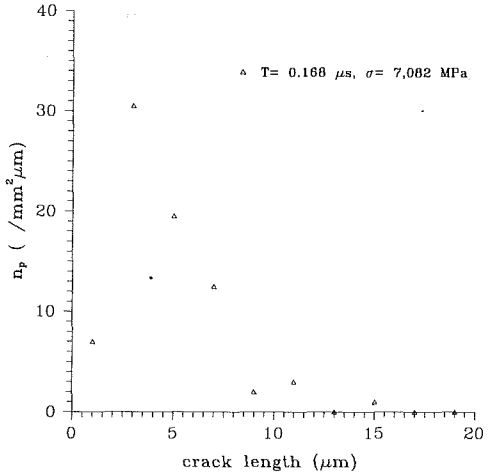

Fig. 4 Typical distribution of microcracks

specimen and loading duration. It is observed that the crack extension Δc of about 10 μm was produced during loading time of about 1 μs. The typical extension rate, therefore, would be 10 μm/μs under the stress pulse loading.

Hence, if the tests for nucleation study are carried out with loading time of about 0.1 μs, the expression (10) leads to

$$c >> \theta \cdot A(c) \cdot \delta t \sim \theta \times 1\mu\text{m}. \tag{11}$$

This is, if the nucleated size of microcracks is several micrometers, one can apply the observed number density of microcracks $n(c, \delta t)$ and expression (9) to obtain the nucleation rate of microcracks $n_N(c)$. Of course, the prerequisite is that the loading time must be submicrosecond.

An experimental method, i.e., the short stress pulse technique, was developed in our laboratory (Shen et al., 1985). A thin metal foil attached to a hollow projectile with low impedance support can create a one-dimensional stress pulse with a submicrosecond duration in target when impact between the target and the foil is conducted by making use of a light gas gun (Fig. 2). In the present study, stress pulses of about 100 ns duration were applied to examine the nucleation of microcracks. Figure 3 gives a picture of microcracks under the loading condition.

All the data listed in this paper were taken from a series of impact tests, in which a 0.1-mm thick nickel flyer strikes a 5-mm thick aluminium target. The details of material and testing procedure are given in the papers of Luo (1988) and Shen et al. (1991). The tested specimens were sectioned and polished carefully. The observations were conducted with an S-570 Scanning Electron Microscope and an Image Analysis System. Particularly, the statistics of microcracks, such as the visual length, orientation, number, etc., can be readily obtained by means of the instruments. Figure 4 shows a typical distribution of microcracks formed in the aluminum alloy target under the stress pulse loading with a duration of about 100 ns. The distribution of microcracks on a polished plane shows the following several distinct features with respect to visual length:

(1) there is a peak in the count at some crack length;
(2) the count tends to zero when crack becomes too long or too short; and
(3) the distribution curve is not symmetrical.

For comparison, the normalized distribution of the number density of microcracks ρ (defined as $\rho_p(c) = n_p(c)/\int_0^\infty n_p(c)dc$) where subscript p denotes the parameters on sectioned surface and also that of second-phase particles, on a polished section of the sample, are shown in Figs. 5 and 6.

The two distributions are qualitatively similar to each other. In addition, the locations of the two peaks in the two curves are in the same range, i.e., 2-5 μm. Furthermore, the value of crack length seems to be reasonable for the requirement for nucleation study, see expression (11). All of these offer corroborative evidence that the observed distribution is a proper representation of the nucleation of microcracks.

The data of the normalized distribution of number density of microcracks ρ can be fitted to Weibull's function as

$$\rho_p(c) \sim c^{m-1} \cdot \exp(-c^m) \tag{12a}$$

or a function similar to Rayleigh's function

$$\rho_p(c) \sim c^m \cdot \exp(-c^2) \tag{12b}$$

(Fig. 5), where subscript p denotes the quantities on a sectioned surface.

4 Law of Nucleation Rate

Before continuing, two points should be made. Since the difference between the number density of microcracks $n(c)$, i.e., the number of cracks in unit physical volume and unit phase space volume, and the corresponding variable on sectioned surface $n_p(c)$ depends on a integration with respect to crack length scale only, the prerequisite to the nucleation study, i.e., formulas (9) and (11) still works for $n_p(c)$. Secondly, we prefer to retain the obtained data on nucleation in its original form, namely the distribution function on a sectioned surface, because all simple transformations of surface counting into

volume distribution are based on some further assumptions on cracks (Seaman et al., 1978). We believe that the original form of nucleation distribution function may be more helpful for further examination.

Now, we are quite convinced that the data obtained by short stress pulse technique are a fair representation of the nucleation of microcracks. However, in practice we need a concise expression of the law of nucleation rate. Then the question is how to determine the expression from obtained data. It has been observed that the cracking is mostly confined to the second-phase copper particles in the aluminum alloy. More importantly, for second-phase particles of all sizes, only part of them became debonded. To look for the stress dependence and size distribution of nucleation of microcracks, we should once more turn to examine the normalized distribution of the number density of microcracks. Figures 5(a) ($\rho_p(c)$) presents the experimental normalized number density of microcracks where n_p is the number density on a sectioned surface and N is the sum of the microcracks. The loading duration ranges mainly from 0.14 μs to 0.17 μs and the stress amplitude from 2.5 to 7.5 GPa. Figures 5(b) and 5(c) present two fittings. We also provide a normalized cumulative measure, i.e., the cumulative number of cracks per unit area divided by the total number of cracks per unit area. This curve shows better fitting, but disguises some scatter and deviations (Figs. 5(b), 5(c), and 5(d)). According to the definition of ρ_p and the approximate solution (8) and (9), we can derive

$$\rho_p(c, t, \sigma) = \frac{n_p(c, t, \sigma)}{\int_0^\infty n_p(c, t, \sigma)dc} \cong \frac{n_{Np}(c, \sigma)\delta t}{N_{Np}(\sigma)\delta t}$$

$$= \frac{n_{Np}(c, \sigma)}{N_{Np}(\sigma)} = \rho_p(c, \sigma), \quad (13)$$

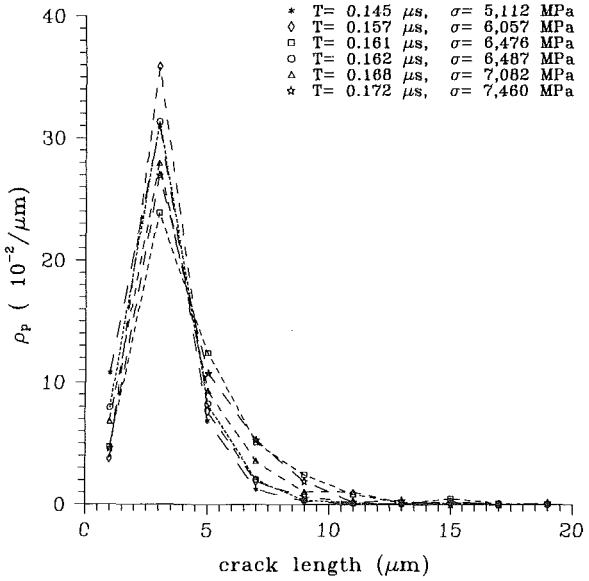

Fig. 5(a) Experimental data

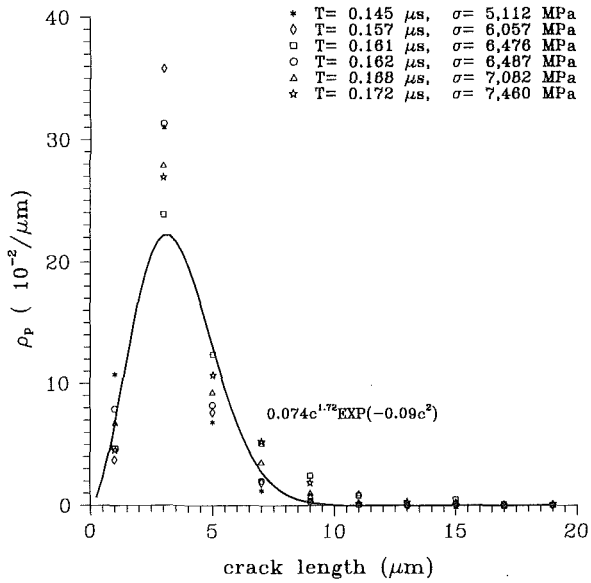

Fig. 5(b) Fitting by $c^m \exp(-Bc^2)$, similar to Rayleigh's distribution

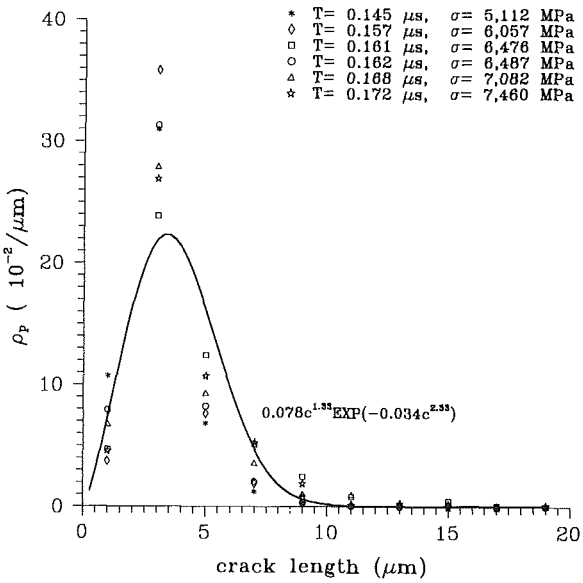

Fig. 5(c) Fitting by Weibull's distribution, $c^{m-1} \exp(-Bc^m)$

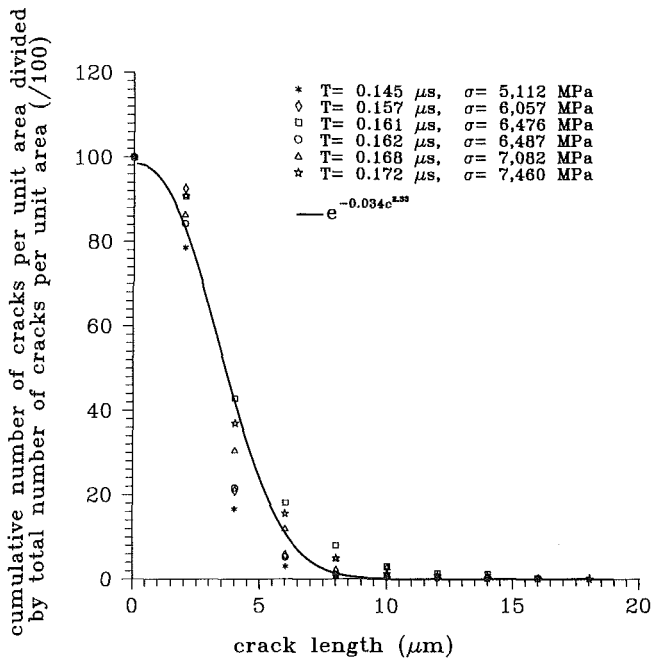

Fig. 5(d) Cumulative size distribution of cracks

Fig. 5 Normalized distribution of number density of microcracks, ρ_p, showing $\rho_p(c, \sigma)$ is insensitive to loading stress, $\rho_p(c, \sigma) \sim \rho_p(c)$

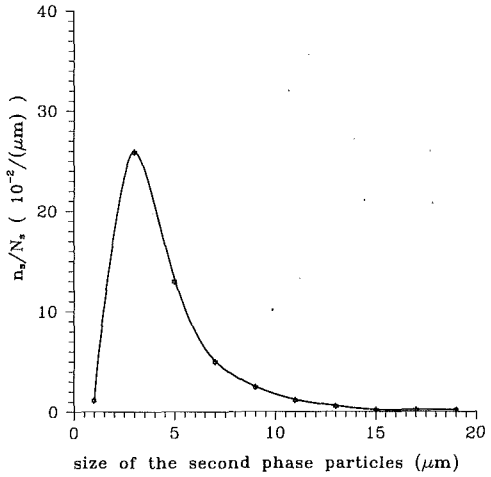

Fig. 6 Distribution of the second-phase particles; n_s : number of the second-phase particles per unit area per unit particle length on the sectioned surface; N_s : total number of the second phase particles per unit area on the sectioned surface

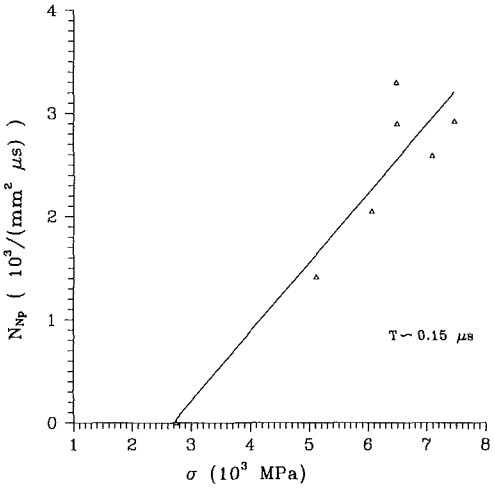

Fig. 7 Relation between the nucleation of microcracks and the loading stress

where

$$N_N(\sigma) = \int_0^\infty n_N(c, \sigma)\, dc. \tag{14}$$

Again, by examining Fig. 5 carefully, one can observe that ρ_p can be expressed as a function of a single variable (crack length c) irrespective of stress σ in the experimental range, namely

$$\rho_p(c, \sigma) \sim \rho_p(c). \tag{15}$$

Therefore, substitution of formula (15) into (13) gives the nucleation rate of microcracks n_{Np}

$$n_{Np}(c, \sigma) = N_{Np}(\sigma) \cdot \rho(c). \tag{16}$$

Furthermore, the data fitting of $N_{Np}(\sigma)$ gives a roughly linear stress dependence. See Fig. 7 ($N_{Np}(\sigma)$)

$$N_{Np}(\sigma) \sim \left(\frac{\sigma}{\sigma_0} - 1\right). \tag{17}$$

According to formulas (12), (13), (16), and (17), one can deduce

$$n_{Np}(c, \sigma) = K_0 \cdot \left(\frac{\sigma}{\sigma_0} - 1\right) \cdot \rho_p(c). \tag{18}$$

From (12) it follows that

$$n_{Np}(\sigma, c) = K \cdot \left(\frac{\sigma}{\sigma_0} - 1\right) \cdot c^{m-1} \exp(-B \cdot c^m) \tag{19a}$$

or

$$n_{Np}(\sigma, c) = K \cdot \left(\frac{\sigma}{\sigma_0} - 1\right) \cdot c^m \cdot exp(-B \cdot c^2) \tag{19b}$$

where K_0 and K are coefficients. For our experimental range σ = (2500 ~ 7500 MPa), t = (0.14 ~ 0.17 μs). These results become

$$n_{Np} = K \cdot \left(\frac{\sigma}{\sigma_0} - 1\right) \left(\frac{c}{c_*}\right)^{m-1} \exp\left(-\left(\frac{c}{c_*}\right)^m\right), \tag{20a}$$

where

$$K = 971 \quad \text{number/(mm}^2 \cdot \mu\text{m} \cdot \mu\text{s)}$$
$$\sigma_0 = 2689 \quad \text{MPa}$$
$$c_* = 4.27 \quad \mu m$$
$$m = 2.33$$

or

$$n_{Np} = K \cdot \left(\frac{\sigma}{\sigma_0} - 1\right) \left(\frac{c}{c^*}\right)^m \exp\left(-\left(\frac{c}{c^*}\right)^2\right)$$

where

$$K = 1042 \quad \text{number/(mm}^2 \cdot \mu\text{m} \cdot \mu\text{s)}$$
$$\sigma_0 = 2689 \quad \text{MPa} \tag{20b}$$
$$c_* = 3.3 \quad \mu m$$
$$m = 1.72.$$

The stress dependence (16) is consistent with macroscopic and empirical cumulative for incipient spallation (Luo, 1988)

$$\left(\frac{\sigma}{450} - 1\right)^{0.97} \Delta t = 1.21 \tag{21}$$

where the stress is in MPa and time is in μs. On the other hand, according to (17) and (18), the integration of solution (8), with respect to crack length c, can give a linear dependence of the total number of microcracks on tensile stress as well as on the loading time (Bai et al., 1991)

$$\left(\frac{\sigma}{\sigma_0} - 1\right) \Delta t \sim N_p, \tag{22}$$

where $N_p = \int_0^\infty n_p\, dc$ is the total number of microcracks over a unit area. Clearly, for incipient spallation, the macroscopic experimental criterion (21) and microscopic theoretical derivation (22) are in good agreement.

5 Disscussion

Complete spallation seems to be a sort of material instability, i.e., the evolution and then abrupt transition into large-scale coalesence of numerous microcracks. To understand this kind of micro-macroscopic material instability, the following preliminary and essential facts have been explored:

1 Based on a general solution to the statistical evolution of a one-dimensional system of ideal microcracks, the initial development of microdamage, under planar impacting load, can be analyzed. Moreover, a prerequisite to experimental investigation of nucleation of microcracks was derived.

2 A short stress pulse technique developed by means of a light gas gun was applied to meet the prerequisite and to obtain the data relevant to the nucleation of microcracks.

3 The normalized number density of microcracks was found to have a asymmetric distribution, which is in agreement with that of second-phase particles.

4 Furthermore, the normalized number density of microcracks shows approximate stress-independence in the experi-

mental range. Therefore, the law of nucleation rate of microcracks can be expressed as a separable function of stress and crack size.

5 The nucleation rate of microcracks was shown, to be, by experimental results, of roughly linear dependence on loading stress.

6 Above all, the nucleation rate of microcracks can be expressed in the form

$$n_{Np} = K \cdot \left(\frac{\sigma}{\sigma_0} - 1\right) \cdot f\left(\frac{c}{c^*}\right) .$$

An illustrative data fitting of the nucleation rate of microcracks on the sectioned surface in an aluminum alloy was given.

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